JKAU: Eng. Sci., **vol. 11 No. 2,** pp. 107-114 (1419 A.H. / 1999 A.D.)

Suboptimal Decentralized Control of Multivariable Systems

ISMAIL A. EL-SHAHAT
Nuclear Materials Authority, P.O. Box 530 Maadi, Cairo, Egypt

ABSTRACT. A state-space design for multivariable, suboptimal decentralized control is presented. It is based upon the scalar multiplier with a cost function $\beta\overset{*}{J}$. A large-scale system consisting of a number of subsystems is used. The suboptimal control law for the large system is obtained from the suboptimal laws of the subsystems. Since the control laws of the subsystems are obtained by solving the low-order matrix Ricatti equations, a considerable saving in computer storage and time is attained. Sufficient conditions for the optimal cost function are also given. The behaviour of the algorithm is illustrated by a simulated example.

KEYWORDS: Optimal control; Suboptimal control; Decentralized systems.

1. Introduction

When the order of the system is very high, the solution of the matrix Riccati equation is required when designing the suboptimal controls for systems with quadratic performance criteria. The number of nonlinear differential equations to be integrated becomes very large, if the order of the system is high. It follows that feedback controller of state information can be proven to be the major cost of the virtual situations such as: power-system control, process control and complex industrial system controls. To overcome this difficulty, many methods mentioned in Meditch[1], Aoki[2], Chidambara and Schanker[3] as well as Isaken and Payne[4] have been proposed to obtain suboptimal controls for higher-order systems. Hassan and Singh[5] developed a procedure for computing under optimal or suboptimal decentralized controller. In Reddy and Rao[6], the suboptimal control law for the large-scale system is derived from the optimal control laws of the subsystems. Since the optimal control laws of the subsystems are obtained by solving the lower-order matrix Riccati equation; Skelton and Xu[7], Yaz[8], Ismail[9], Ismail[10], Ismail[11] and Toivonen and Makila[12] have developed an entirely new approach to the solution of the decentralized suboptimal control problem. In this research paper, a new design procedure, which is motivated by its simplicity and the fact that the algorithm is based on sub-optimality multiplier $\overset{*}{\beta}J$ is developed. It uses the well-known standard Ricatti equation when reliable conditions are satisfied to form a decentralized output feedback control gains. Numerical results show that the algorithm has numerous advantages.

2. Control Problem

Research in decentralized control has been motivated by using conventional modern control theory as state space equations. In this research paper, a large-scale system consisting of N subsystem that can be defined for a decentralized system is used. The suboptimal problem of a large-scale system consisting of N subsystem, with βJ for the cost function, can be written as

$$J = \sum_{K=1}^{N} \int_{0}^{\infty} e^{2\alpha}(X_K^T Q_K X_K + U_K^T R_K U_K)\ dt \quad \alpha \geq 0 \tag{1}$$

Where α is a real scalar, $Q_K \geq 0$ and $R_K > 0$. J in equation (1) is minimized according to the linear, time-invariant, continuous system, consisting of N subsystems as follows:

$$\begin{aligned} \dot{X}_K &= A_K X_K + B_K U_K \quad , X_K(0) = X_0 \\ y_K &= C_K X_K \quad , K = 1, \ldots , N \end{aligned} \tag{2}$$

where K is a subsystem,
X_K is a state vector of dimension n_K ,
U_K is a control vector of dimension p_K ,
y_K is an output vector of dimension r_K, and
A_K, B_K and C_k are matrices of ranks n_k, p_k and r_k respectively.

It is required to determine a suboptimal decentralized control law of the form:

$$\overset{*}{U}_K = - F_K y_K \quad , K = 1, \ldots , N \tag{3}$$

where F_K is of feedback gain matrix of dimension r_K

This function will minimize the cost function of the equation (1).

3. Solution of the Problem

A suboptimal decentralized control $\overset{*}{U}_K$ and its cost $\overset{*}{J}$ for the kth subsystem are given by:

$$\overset{*}{U}_K = - F_K C_K X_K = - R_k^{-1} B_K^T \tilde{P}_K X_K \tag{4}$$

$$\overset{*}{J} = X_K^T(0) \tilde{P}_K X_K(0) \tag{5}$$

For evaluating the suboptimal output feedback system, P_K and A_K^n are defined as:

$$\tilde{P}_K = \beta P_K \tag{6}$$

and $$A_K^n \overset{\Delta}{\equiv} A_K + B_K R_k^{-1} B_K^T \tilde{P}_K - B_K F_K C_K \tag{7}$$

where $P_k = P_K^T$ is the unique positive solution of the algebraic Riccati equation:

$$P_K (A_K + \alpha I) + (A_K + \alpha I)^T P_K + Q_K - P_K B_K R_K^{-1} B_K^T P_K = 0 \tag{8}$$

for a given $\beta \geq 1$, if there exists an F_k which satisfies

$$\tilde{P}_K (A_K^n + \alpha I) + (A_K^n + \alpha I)^T \tilde{P}_K + Q_K - \tilde{P}_K B_K R_K^{-1} B_K^T \tilde{P}_K = 0 \tag{9}$$

and $$C_K^T F_K^T R_K F_K C_K \leq \tilde{P}_K B_k R_K^{-1} B_K^T \tilde{P}_K \tag{10}$$

In this case, there exists a suboptimal decentralized output feedback control of equation (4), with which the subsystem (2) yields the minimized cost function of equation (1).

4. Summary of the Algorithm

The proposed algorithm was tested on one system, and the minimum a priori knowledge required by the suboptimal decentralized control is identified. The constraints and the procedures of the algorithms are as follows:

1 – $\alpha \geq 0$ and $\beta \geq 1$ are real scalars,

2 – $F_K \in \theta_K$ is chosen,

where $\theta_K = F_K \in R^{p.r} : A_K + B_K F_K C_K$ is asymptotically stable, and j = 1, where j = number of iterations,

3 – $F_K^J \in \theta_K$ is found, so that:

$$\tilde{P}_K^J (A_k^n + \alpha I) + (A_K^n + \alpha I)^T + \tilde{P}_K^J + Q_K - \tilde{P}_K^J B_K R_K^{-1} B_K^T \tilde{P}_K^T = 0 ,$$

and $$C_K^T F_K^T R_K F_K C_K \leq \tilde{P}_K^J B_K R_K^{-1} B_K^T \tilde{P}_K^J .$$

Consequently, equations (9) and (10) are satisfied. They represent the sufficient conditions for the suboptimal decentralized control.

4 – j : = J + 1 is updated

5 – Then, go to No. 2.

5. A Numerical Example

A control system consisting of three subsystems is considered. It is defined by the following matrices (Yu and Siggers[13], Yu and Moussa[14]:

$$A_{11} = \begin{bmatrix} -0.922 & 1.00 & -2.06 & -0.099 \\ -2.750 & -2.78 & -1.36 & -0.037 \\ 0.000 & 0.00 & 0.00 & 0.000 \\ -4.950 & 0.00 & -5.55 & -0.039 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.024 & 0.0 & -0.087 & 0.002 \\ -0.158 & 0.0 & 1.110 & -0.001 \\ 0.000 & 0.0 & 0.000 & 0.000 \\ 0.222 & 0.0 & 8.170 & 0.004 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 0.072 & 0.0 & -0.25 & 0.003 \\ 0.460 & 0.0 & 2.80 & -0.020 \\ 0.000 & 0.0 & 0.00 & 0.000 \\ 0.924 & 0.0 & 1.75 & 0.020 \end{bmatrix}, B_1 = \begin{bmatrix} 0.0 & 0.4 \\ 0.0 & 0.9 \\ 0.8 & 0.4 \\ 0.0 & 09 \end{bmatrix}, \; C_1 = I$$

$$A_{21} = \begin{bmatrix} 0.21 & 0.0 & 0.121 & 0.003 \\ -1.10 & 0.0 & -0.620 & -0.015 \\ 0.00 & 0.0 & 0.000 & 0.000 \\ 0.92 & 0.0 & 1.750 & 0.020 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.21 & 1.0 & 1.60 & -0.005 \\ -1.90 & -1.8 & 9.30 & -0.120 \\ 0.00 & 0.0 & 0.05 & 1.000 \\ -3.10 & 0.0 & -5.55 & -0.032 \end{bmatrix}$$

$$A_{23} = \begin{bmatrix} 0.60 & 0.0 & 0.46 & 0.002 \\ -1.00 & 0.0 & 1.49 & -0.040 \\ 0.00 & 0.00 & 0.00 & 0.000 \\ 0.12 & 0.0 & 2.98 & -0.280 \end{bmatrix}, B_2 = \begin{bmatrix} 0.9 & 0.9 \\ 0.0 & 0.0 \\ 0.8 & 0.4 \\ 0.0 & 0.9 \end{bmatrix}, \; C_2 = I$$

$$A_{31} = \begin{bmatrix} -0.002 & 0.0 & 0.089 & 0.00 \\ -6.780 & 0.0 & -1.100 & -0.09 \\ 0.000 & 0.0 & 0.000 & 0.00 \\ -1.240 & 0.0 & 2.980 & -0.28 \end{bmatrix}, A_{32} = \begin{bmatrix} 0.01 & 0.0 & 0.22 & 0.000 \\ -2.10 & 0.0 & 1.70 & 0.123 \\ 0.00 & 0.0 & 0.00 & 0.000 \\ 0.07 & 0.0 & 0.37 & 0.011 \end{bmatrix}$$

$$A_{33} = \begin{bmatrix} -0.197 & 1.0 & -1.20 & -0.003 \\ -5.44 & 2.0 & 7.00 & -2.370 \\ 0.0 & 0.0 & 0.05 & 1.000 \\ -3.4 & 0.0 & -2.10 & -0.017 \end{bmatrix}, B_3 = \begin{bmatrix} 0.8 & 0.4 \\ 0.0 & 0.9 \\ 0.8 & 0.4 \\ 0.0 & 0.9 \end{bmatrix}, \; C_3 = I$$

where Q = I, R = I, P = I, and I is a unity matrix.

The suboptimal decentralized feedback of this example is carried out through the use of computer program devised for the present algorithm which depends on the minimization of the cost function $\beta \overset{*}{J}$. The results of the computations are tabulated hereafter for the three subsystems stated in Table (1). It is observed from Table (1), with the suboptimal controls laws of the subsystems used in the example, that the algorithm can be applied to a large-scale linear systems. Besides, the step response of the suboptimal closed-loop was determined as shown in Fig. (1). From the four curves, it is clear that the states of the variables (VAR_1, VAR_2, VAR_3, VAR_4) for $subsystem_1$, $subsystem_2$ and $subsystem_3$ reached the steady state in a short time.

TABLE 1. Results of the algorithm applied to the example.

	Subsystem_1	Subsystem_2	Subsystem_3
$\overset{m}{A}$	$\begin{bmatrix} 1.80 & -3.8 & -2.3 & 0.0 \\ 0.00 & -2.9 & 0.0 & 0.0 \\ 1.21 & 0.0 & -2.3 & 0.0 \\ -1.36 & 0.0 & 0.0 & 0.0 \end{bmatrix}$	$\begin{bmatrix} -7.1 & -2.6 & 7.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ -19.2 & 0.0 & 6.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$	$\begin{bmatrix} -1.0 & -4.5 & -1.0 & 0.0 \\ 0.0 & -2.7 & 0.0 & 0.0 \\ 0.9 & 0.0 & -1.1 & 0.0 \\ 2.3 & 0.0 & 0.0 & 0.0 \end{bmatrix}$
$\tilde{P}$	$\begin{bmatrix} 10.5 & 8.4 & -4.6 & 0.0 \\ 1.0 & 1.0 & -1.0 & 0.0 \\ -7.9 & 0.0 & -4.9 & 0.0 \\ -2.7 & 0.0 & -1.4 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 13.4 & 12.0 & 1.2 & 0.0 \\ 0.0 & 1.0 & -0.1 & 0.0 \\ 0.6 & 0.0 & 3.9 & 0.0 \\ -0.9 & 0.0 & 1.5 & 1.0 \end{bmatrix}$	$\begin{bmatrix} 12.4 & 11.2 & -1.7 & 0.0 \\ 0.0 & 1.0 & -0.10 & 0.0 \\ -1.5 & 0.0 & 3.60 & 0.0 \\ -6.7 & 0.0 & -1.10 & 0.0 \end{bmatrix}$
F	$\begin{bmatrix} 2.1 & 6.7 & 0.0 & -7.6 \\ -0.5 & 4.3 & 0.0 & -5.1 \end{bmatrix}$	$\begin{bmatrix} 12.5 & 9.6 & 0.0 & 4.3 \\ 11.4 & 5.7 & 0.0 & 4.1 \end{bmatrix}$	$\begin{bmatrix} 8.7 & 8.9 & 0.0 & 1.6 \\ -1.8 & 5.4 & 0.0 & 0.3 \end{bmatrix}$
$\overset{m}{A} + BFC$	$\begin{bmatrix} -5.1 & -1.0 & 2.6 & 0.1 \\ 2.7 & -3.2 & 1.4 & 0.0 \\ 0.0 & 0.0 & -6.0 & 0.0 \\ 4.9 & 0.0 & 5.5 & -5.9 \end{bmatrix}$	$\begin{bmatrix} -6.60 & 0.0 & -0.5 & -0.1 \\ 1.00 & -6.0 & -1.5 & 0.1 \\ 0.00 & 0.0 & -6.0 & 0.0 \\ -0.12 & 0.0 & -2.9 & -5.7 \end{bmatrix}$	$\begin{bmatrix} -5.9 & 0.0 & -0.1 & 0.0 \\ 6.8 & -6.0 & 1.1 & 0.1 \\ 0.0 & 0.0 & -6.0 & 0.0 \\ 1.2 & 0.0 & -3.0 & -5.7 \end{bmatrix}$
α	1.000	1.000	1.000
β	1.500	1.500	1.500
$\overset{*}{J}$	34.519	32.632	17.022

Conclusions

An output feedback controller of suboptimality degree β is applied to a large-scale linear system to solve the suboptimal decentralized control problem. The algorithm is based on a standard Riccati equation, and is applicable when certain conditions are satisfied to form a suboptimal decentralized output feedback control gain. The developed suboptimal decentralized controller is easy to implement, simple to iterate (low number of iterations), requires insignificant computation time and does not require large memory. This has been illustrated by an example.

Acknowledgement

The author would like to express his indebtedness to Prof. Ahmed A. Ammar, Nuclear Materials Authority, Cairo, Egypt for revising linguistically the manuscript.

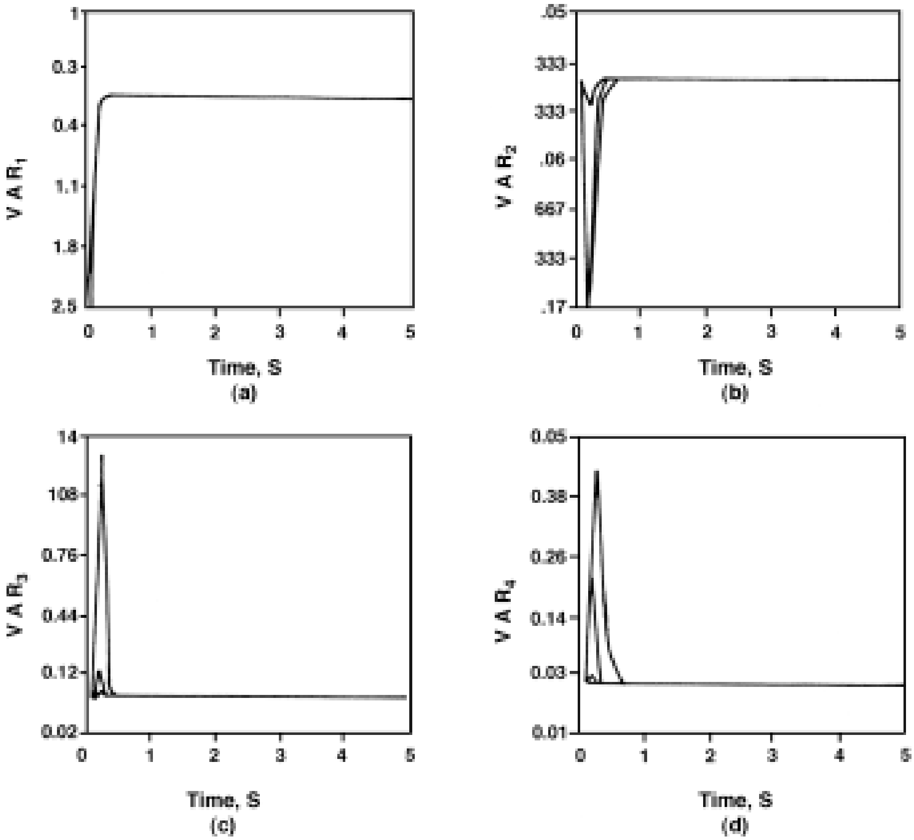

FIG. 1. Suboptimal responses of (a) VAR_1, (b) VAR_2, (c) VAR_3 and (d) VAR_4 for the three subsystems.

References

[1] **Meditch, J.S.,** A class of suboptimal linear controls, *IEEE Trans. on AC-11,* pp. 4 33-439 (1966).

[2] **Aoki, M.,** Control of large-scale dynamic systems by aggregation, *IEEE Trans. on AC-13,* pp. 246-253 (1968).

[3] **Chidambara, M.R.** and **Schanker, R.B.,** Lower order generalized aggregate model and suboptimal control, *IEEE Trans. on AC-16,* pp. 175-180 (1971).

[4] **Isaken, L.** and **Payne, J.,** Suboptimal control of linear system by augmentation with application to free way traffic regulation, *IEEE Trans. on AC-18,* pp. 210-219 (1973).

[5] **Hassan, M.** and **Singh, M.G.,** A hierarchical computational structure for near optimal decentralized control. *IEEE Trans. on SMC-8,* pp. 429-432 (1978).

[6] **Reddy, R.L.** and **Rao, B.S.,** Control of a large linear system through system subdivision, *Proc. IEE,* **124**(8): 725-728 (1977).

[7] **Skelton, R.E.** and **Xu, J.H.,** Output feedback controllers of suboptimality degree B, *IEEE Trans. on AC-35,* pp. 1369-1373 (1990).

[8] **Yaz, E.,** General suboptimal approach to the control and estimation of discrete-time systems, *Int. J. System, SCI-16,* pp. 1095-1108 (1985).

[9] **Ismail, A.E.,** Decentralized suboptimal controller, *AMSE Int. Conf.,* **1:** 11-17 (1992).

[10] **Ismail, A.E.,** Decentralized optimal controller, *10th National Radio Science Conference,* **H4:** 1-8 (1993).

[11] **Ismail, A.E.,** Decentralized Self-Tuning Controller of Sub-optimality B, *Arab Gulf J. Scient.Res.,* **17**(1): 49-57 (1999).

[12] **Toivonen, H.T.** and **Makila, P.M.,** A descent Anderson Moor algorithm for optimal decentralized control, *Automatica,* **21**(6): 743-744 (1985).

[13] **Yu, Y.N.** and **Siggers, C.,** Stabilization and optimal control signals for a power system, *IEEE Trans.,* ***AC*-PAS-90:** 1469-1481 (1971).

[14] **Yu, Y. N.** and **Moussa, H.M.,** Optimal stabilization of multi-machine system, *IEEE Trans.on AC-17,* pp. 1174-1182 (1972).

حـاكم لامركزي شبـه مثالي للأنظمة الخطيـة متعـددة المتغيـرات

اسماعيل الشحات

هيئة المواد النووية – المعادي – القاهرة – جمهورية مصر العربية

المستخلص . يبرز هذا البحث تصـميماً لحاكم لامركزي شـبه مثالي في مـجال مـعين للأنظـمة الخطـية مـتعددة المتغيـرات . ينبني هذا الحـاكم على أساس مـضاعف عددي ذي دالـة تكلفة (ßJ*) بهدف الحـصول على أنسب تكلفة مـثلي للنظام . تم استخدام نظام ذي نطاق واسع يتـألف من عدد من الأنظمة الفـرعية . تم احراز قـاعدة حاكمـة شبه مثـالية للنظام الواسع ككل من القـواعد شبـه المثالـية للأنظمـة الفرعـية . حيـث إنه يمكن صيـاغة قـواعد ضـابطة لـلأنظمـة شـبـه المثـاليـة عن طريق حل مـعـادلات ريكاتي ذات المصفوفات من الرتب الدنـيا ، فإنه يمكن الحصول على وفر كبير في السعة ووقت الحاسب الآلي . تم كـذلك توفير ظروف كـافية لتكلفـة شبه مثـالية ، كما تم تبيان سلوك الخوارزمية وفقًا لمثال محاكي .